Methods for Establishing Confidence Bounds on Sector Demand Forecasts

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Abstract

Prediction of sector demand, which is usually measured in terms of the number of aircraft, is essential for air traffic management since controller actions such as handoffs and communications, and the need for monitoring directly scale with the number of aircraft in the airspace. This forecasting of traffic-counts is routinely done using a trajectory prediction method in the Enhanced Traffic Management System. The computed demand data is compared against pre-established values, known as monitor alert parameters, to determine if flow restrictions are needed. This paper first examines traffic-count forecasting using a deterministic procedure implemented in the Future ATM Concepts Evaluation Tool. Based on past analysis of Enhanced Traffic Management System data quality, some aircraft were removed from the recorded data during the preprocessing step to improve the quality of the data. Steps were also taken to accommodate data drops, correct reported altitude errors, and estimate groundspeed. Traffic-count predictions were generated for twenty sectors in fourteen Air Route Traffic Control Centers in the continental United States at prediction intervals from 15 minutes through two hours. Prediction error (difference between the predicted and actual traffic-counts) statistics are presented for these predictions, which provide confidence bounds on the predicted traffic-counts. Prediction accuracy is also examined in terms of percentages of correct-prediction, under-prediction and over-prediction as a function of prediction intervals. These results for the twenty sectors are correlated to the decreasing percentages of airborne aircraft as a function of increasing prediction intervals. In the second part of the paper, probabilistic prediction of traffic-counts is examined. The general expression for computing the probability of having a certain number of aircraft or more in the sector at any given time is derived. Expressions are given for computing the probability of an aircraft being in a sector by assuming a Gaussian model of departure uncertainty. The procedure for computing the probabilities of exceeding pre-established traffic-count thresholds and their use in decision making is illustrated via a numerical example. Both the deterministic and the probabilistic traffic-count forecasting procedures provide a means for establishing confidence bounds, which are useful from decision making point-of-view.

1. Introduction

The central purpose of traffic flow management is to respond to demand-capacity imbalances so that the air traffic in the United States continues to flow smoothly. Of the several capacity constrained resources, the significant ones are airports, fixes and sectors. Airports are capacity constrained due to fixed number of runways and their availability depending on wind and visibility conditions. Capacities of fixes are limited by air traffic controller workload when several aircraft are held or metered at the fixes because of landing capacity limitations at the airport. Anytime an aircraft is put into a holding pattern, the controller needs to frequently communicate with the pilot to determine the amount of time the aircraft can stay airborne, especially after a long-duration flight when the aircraft is low on fuel. Like the fix capacities, sector capacities are also limited by controller workload considerations. There are several factors that effect controller's perception of workload in a sector including the number of aircraft in the sector, the proportion of climbing, cruising and descending aircraft, mix of jet and turboprop traffic, airway layout and sector geometry [1]. Of these factors, number of aircraft is generally accepted to be a key factor because actions such as handoffs, communications and monitoring directly scale up with the number of aircraft being controlled. Due to these reasons, an accurate forecast of the number of aircraft, which represents demand for the available sector capacity, is essential for a decision support system designed to mitigate demand-capacity imbalances.

Deterministic forecasting of number of aircraft in sectors, fixes and airports is routinely done within the Enhanced Traffic Management System (ETMS), which

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is used operationally by both the Traffic Management Units within the Air Route Traffic Control Centers (ARTCC's) and the Air Traffic Control System Command Center (ATCSCC). The forecasting method employed in ETMS relies on computation of each aircraft's entry and exit times at each sector along the path of flight, and the times of arrival at fixes and airports. Sector entry and exit times are then used for counting the expected number of aircraft in the sector as a function of time. This demand data is then compared against pre-established Monitor Alert Parameter (MAP) for the sector to determine if flow restrictions are needed [2].

Algorithms for predicting the position of aircraft or entry and exit times assume that the departure time, route of flight, trajectory profile, weather and flow restrictions are known accurately. This assumption is often invalid because of uncertainties. A certain percentage of aircraft depart later than their scheduled and filed departure time due to baggage handling, mechanical difficulties, passengers, visibility and weather conditions, departure sequencing, and traffic flow management initiatives (for example, ground stop and ground delay). Similarly, the aircraft may not follow the initially filed route of flight, which is used for trajectory prediction, because of rerouting due to weather conditions encountered enroute. The actual trajectory of the aircraft may also significantly differ from the trajectory profile assumed for the type of the aircraft. Wind profile, forecast based on measurement data and mathematical models, used for trajectory prediction may be quite different from the actual wind profile. Another source of error is the open-loop nature of prediction that does not take traffic flow restrictions into account. Part of the difficulty is that it is not quite straightforward to model the impact of enroute restrictions on the flight of a single aircraft. For example, miles-in-trail restriction can only be defined with respect to the aircraft in the front. Thus, the effect of such a restriction is that the arrival sequence at a downstream location affects the departure sequence at an upstream location and consequently the 4-dimensional trajectory (locations as a function of time) of a single aircraft cannot be predicted independently of the trajectories of other aircraft. Prediction difficulty is further compounded with restrictions at multiple locations.

Given that open-loop traffic-count predictions do not match the actual traffic-counts, estimates of uncertainty bounds about these predictions are needed to aid decision-making. Two different approaches are explored in this paper for establishing confidence bounds on the traffic-count forecasts in the sectors. The first method employs a deterministic procedure for predicting traffic counts and then uses historical prediction error (with respect to actual traffic-counts) statistics for establishing uncertainty bounds. The second method uses a probabilistic approach to provide the probability of a certain number of aircraft or more being in the sector at specified instants of time. Thus, this method also provides an alternative representation of confidence bounds. An example of this representation is provided using departure time uncertainty in the probabilistic forecasting method.

There have been suggestions to overcome the inaccuracy introduced into trajectory prediction due to departure time uncertainty by including departure uncertainty distributions in a stochastic forecasting method [3, 4]. The focus on departure time uncertainty is largely guided by the assumption that departure delay uncertainty is the main contributor to the trajectory prediction errors [5]. Departure uncertainty has been considered as a special case of a more general sector entry time and exit time uncertainties in this paper. The main contribution of the paper is extension of the deterministic procedure to a probabilistic one for predicting traffic-counts in the sectors and then using the resulting probabilities for decision-making.

The paper is organized as follows. Section 2 examines the deterministic procedure for predicting traffic-counts in sectors and the associated confidence bounds as a function of prediction time intervals. The technique for probabilistic prediction of traffic-counts is developed in Section 3. The general expression for computing the probability that there will a certain number of aircraft or more (for example, ten or more) at any given time in the future in a sector is derived in Section 3. Sector entry and exit time uncertainties resulting from departure uncertainty are used for computing probability of individual aircraft being in a sector at a given instant of time in Section 4. A numerical example is presented in Section 4 to illustrate the procedure of using the probabilities of individual aircraft and the expressions in Section 3 for computing the probabilities of exceeding pre-established traffic-count thresholds in sectors. The paper is concluded in Section 5.

2. Deterministic Prediction of Traffic-Counts

The process of prediction of the number of aircraft in a sector as a function of time consists of two steps: 1) predicting the location of every aircraft (both currently airborne and proposed to depart in the future) as a function of time, and 2) counting the number of aircraft that fall within the boundaries of the sector.

Most algorithms for trajectory prediction require the locations of the origin and the destination, a time of departure, aircraft performance data (thrust, drag and lift data or vertical and horizontal speed profiles), wind velocity profile and a route of flight. The proposed time of departure and the route of flight as defined by the flight plan, current location of the aircraft derived from the radar, groundspeed, climb/descent rate and heading angle are available in the ETMS and also provided as output data in several different formats. In addition to the data received from ETMS, databases are needed for performance models (consisting of thrust, drag and lift data or vertical and horizontal speed profiles) tailored to the specific aircraft types. If thrust, drag and lift data are available, the linear acceleration and rate of change of flight path angle are computed and integrated to determine the airmass-relative velocity and flight path angle. They are then used along with the wind velocity components and the course angle, specified by the route of flight, to predict the position of the aircraft. The three-dimensional point-mass equations of motion that use the thrust, drag and lift data are given in Reference 6.

An alternative procedure is used, if climb/descent rates and airspeed are given as a function of altitude, instead of thrust, drag and lift data. The climb and descent rates are integrated forward to obtain the altitude at the next instant of time. Groundspeed is obtained using the horizontal components of the airmass-relative velocity and the wind velocity along the path specified by the course angle. The groundspeed and the course angle are then used in the equations of motion to predict the location of the aircraft. This method is implemented in the Future Air Traffic Management Concepts Evaluation Tool (FACET), which has been used for generating all the results described here. The details of computation of groundspeed and course angle in a wind field and their use in the equations of motion are described in Reference 7. Latitude, longitude and geometric altitude are obtained by integrating these equations. Additional description of this trajectory prediction method is provided in Reference 8.

Two main issues plague the trajectory prediction process. The first issue relates to the quality of data that is input to a trajectory prediction algorithm. The study in Reference 9 investigated the quality of the air traffic data provided by ETMS from trajectory prediction accuracy point-of-view. Data quality was examined in terms of the availability of flight plans, deviations from flight plans, departure delays, accuracy of altitude data, and the extent of data drops. Based on examination of one day’s traffic, this study concluded that on an average, flight plans were missing for 12% of the aircraft in the airspace at any given time. In the worst case, flight plans were missing for 19% of the aircraft. It was found that 88% of the aircraft stay within ten miles from their filed flight plans. About 8% stay within 10-20 miles and 3% stay within 20-50 miles. Only 1% of the aircraft deviate by more than 50 miles. Analysis of departure delays revealed that about 92% of the aircraft depart within ± 20 minutes. The accuracy of altitude data output via ETMS was found to be poor during the climb and descent segments. Data drops were found to be a significant data quality issue. On an average, track data are unavailable for 37% of the aircraft at any given instant of time. In order to compensate for all the data quality issues analyzed in Reference 9, certain heuristics have to be added to the basic trajectory prediction algorithms.

The second important issue relates to evaluation of the accuracy of traffic-counts based on predicted trajectories. The most prevalent technique is to compare the predicted traffic-counts against the actual number of aircraft in the sector obtained from recorded air traffic data [10-12]. This approach suffers from the fact that trajectory prediction is done without knowledge of prior traffic flow control actions while the actual number of aircraft in a sector at any given time is an outcome of traffic flow control actions in response to the forecast [5]. Thus, in cases where control actions are taken, the predicted traffic-counts cannot be compared with the actual traffic-counts [5]. Comparisons are possible in those cases where both the predicted traffic-counts and the actual traffic-counts are significantly below the MAP value, based on the hypothesis that in these instances control actions were not taken.

In summary, due to data quality and evaluation issues, consistent bookkeeping of the aircraft that are predicted to be in a sector and that actually fly through the sector is needed. For example, if flight plan is missing for an aircraft when prediction is made, that aircraft should be excluded from the actual count at the time where comparison is desired. Similarly, if an aircraft deviates by more than a specified distance from the flight plan, that aircraft should not be included in the actual count. The main benefit of this approach is that it allows one to assess the quality of predicted data when the prediction algorithm has all the information it needs. A comparison of the predicted data against actual data without these adjustments reflects more on the quality of the input data rather than the ability of the prediction algorithm to predict correctly.

To enable comparison of the traffic-counts generated using the trajectory prediction algorithm implemented in FACET with the actual traffic-counts in the sectors, ETMS data were collected for 24 hours on July 17, 2002. Data in “*Orig*” format (one of the several formats in which ETMS data are provided), consisting of messages for proposed departure time, cancellation, filed flight plan, amended flight plan, actual time of departure, track position and actual time of arrival, were first analyzed using FACET. Results of the data quality analysis for this data are reported in Reference 9. In light of the data quality analysis, some aircraft were removed from the data to improve data quality. The resulting data only included aircraft that originated and landed within the confines of the Continental U. S. airspace. Aircraft that deviated by more than 30 miles from their flight plans at any time during their flight were removed. Aircraft that departed earlier or later than ± 40 minutes with respect to their proposed departure times were also removed.

In addition to data conditioning steps taken during the pre-processing phase, three additional corrective actions are taken during the trajectory prediction process. The first action is taken to safeguard against data drops by retaining aircraft position data for a period of time and by estimating the position of aircraft at time instants when data drops occur. Track and flight plan information for an aircraft is preserved in FACET for 20 minutes unless an arrival message is provided by ETMS. Track and flight plan data are removed if the last reported track position is near the destination airport and the aircraft is expected to have landed. If the aircraft is far away from its destination, its position is estimated by integrating the trajectory forward from the previous track position to the next waypoint along the flight plan. Situations where flight plans are unavailable, position is estimated assuming that the aircraft maintain course. In addition to assumptions about path, FACET assumes constant groundspeed and altitude for the duration of data drop. The second action is taken to correct for errors in ETMS reported altitude during the climb and descent phases of flight. The heuristic algorithm described in Reference 9 is used for this purpose. Finally, the third corrective action consists of using ETMS reported groundspeed during the cruise phase rather than the groundspeed obtained from aircraft performance models. The procedure consists of using ETMS provided groundspeed (originally computed by the ARTCC Host computer using surveillance data derived from Air Traffic Control Radar Beacon System) and the predicted wind velocity to estimate the airspeed at the initial location. This airspeed is then added to the predicted windspeed to generate an estimate of groundspeed along the path specified by the flight plan. The resulting groundspeed is used in the equations of motion to predict the location of aircraft at future instants of time.

After conditioning the input data and enhancing the trajectory prediction process, traffic-count prediction data were generated for twenty sectors from fourteen ARTCCs listed in Table 1. All the sectors listed in this table are high-altitude sectors that often experience heavy traffic loads.

Table 1: Sectors for which prediction data were generated.

ARTCC	Sectors
Albuquerque	ZAB70
Atlanta	ZTL15
Boston	ZBW10
Cleveland	ZOB29
Denver	ZDV18
Fort Worth	ZFW48, ZFW93
Houston	ZHU26
Indianapolis	ZID84, ZID98
Kansas City	ZKC84, ZKC98
Los Angeles	ZLA30, ZLA35
Miami	ZMA59
New York	ZNY10, ZNY42
Seattle	ZSE48
Washington DC	ZDC12, ZDC72

Figure 1 shows the predicted and the actual time histories of the number of aircraft in Sector 93 (ZFW93 in Table 1) of the Fort Worth ARTCC. The prediction interval of two-hours was used for generating these results. Predicting traffic-counts once every minute two hours into the future generated the predicted time history. The MAP value of 18 aircraft for Sector 93 is marked in the Figure. Observe from the figure that both the actual traffic-counts and the predicted traffic-counts remain below the MAP value.

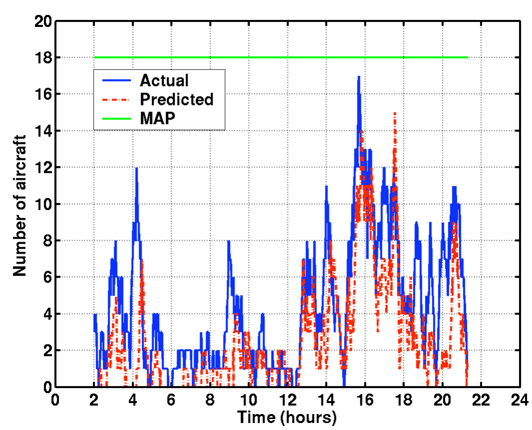

Figure 1: Two-hour prediction and actual traffic-count time histories in Sector 93 of the Fort Worth ARTCC.

Since traffic flow management decisions are made by comparing the peak number of aircraft in a fifteen-minute interval with the MAP value (see: Reference 2), Figure 2 was generated to compare the predicted peak number of aircraft in fifteen-minute intervals with the actual peak traffic-counts. Actual peak traffic-counts were obtained by choosing the maximum number of aircraft that were in the sector in non-overlapping fifteen minute intervals spanning the two to 21 hour time periods shown along the abscissa in Figure 2. Similarly, the predicted peak traffic-counts were obtained from the set of two-hour predictions (once every minute) in each fifteen-minute interval.

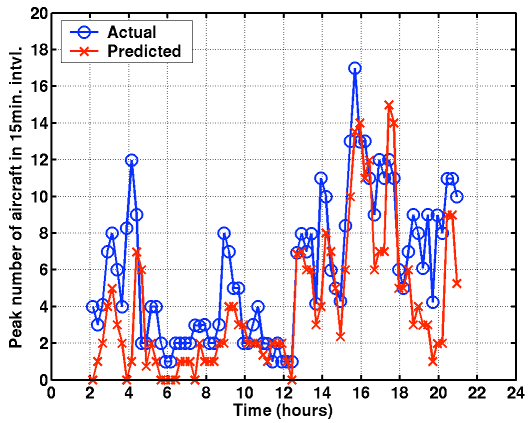

Figure 2: Comparison of two-hour predictions of peak traffic-counts with actual peak traffic-counts in Sector 93 of Fort Worth ARTCC.

The degree of prediction accuracy seen in both Figures 1 and 2 were also seen for the other sectors listed in Table 1. Statistics of the prediction errors (difference between the predicted aircraft-count and the actual aircraft-count in the sectors) were computed for 15-minute, 30-minute, 45-minute, 60-minute and two-hour prediction intervals. Note that these 15-minute through two-hour prediction error samples were available at one-minute intervals (see: Figure 1, which shows two-hour predictions at one-minute intervals). Table 2 shows the 60-minute prediction error statistics. The first column lists the name of the sector. The second and third columns show the average error μ and the standard deviation of the error σ rounded to integer number of aircraft. The fourth and the fifth columns show the average error μ_p and the standard deviation of the error σ_p of peak traffic-counts (see: Figure 2, which shows both predicted and actual peak traffic-counts). The values in these last two columns are also rounded off to an integer number of aircraft. Observe from Table 2 that the average error stays bounded between –2 and +2 aircraft (see: column 2) and the standard deviation stays bounded between 1 and 3 aircraft (see: column 3). The table also shows that the trends in columns four and five for prediction errors of 15-minute peak traffic-counts are very similar to those in columns two and three.

The statistics of the two-hour prediction errors are summarized in Table 3. Comparing the values in Table 3 with those in Table 2, it is seen that the average values μ and μ_p are lower for two-hour predictions, which indicates a tendency to under-predict the traffic-counts. The standard deviation values in columns three and five do not change significantly for two-hour predictions compared to one-hour predictions.

Table 2: Sixty-minute prediction error statistics.

Sector	μ	σ	μ_p	σ_p
ZAB70	0	1	0	1
ZBW10	1	2	1	2
ZDC12	1	2	1	2
ZDC72	2	3	2	3
ZDV18	1	2	1	2
ZFW48	0	2	1	2
ZFW93	0	2	0	2
ZHU26	0	1	0	2
ZID84	0	2	-1	2
ZID98	2	2	3	2
ZKC84	1	2	1	2
ZKC98	1	2	1	2
ZLA30	0	2	0	2
ZLA35	-1	2	-1	2
ZMA59	-2	3	-2	3
ZNY10	2	3	3	3
ZNY42	0	2	1	2
ZOB29	2	3	2	3
ZSE48	0	1	0	1
ZTL15	2	3	2	3

Table 3: Two-hour prediction error statistics.

Sector	μ	σ	μ_p	σ_p
ZAB70	0	2	0	1
ZBW10	0	2	0	2
ZDC12	0	2	0	2
ZDC72	0	3	0	3
ZDV18	0	2	0	2
ZFW48	-1	2	-2	2
ZFW93	-1	2	-1	2
ZHU26	0	2	0	2
ZID84	-1	2	-2	2
ZID98	1	2	2	2
ZKC84	0	2	0	2
ZKC98	0	2	1	2
ZLA30	-1	2	-1	2
ZLA35	-1	2	-2	2
ZMA59	-2	3	-2	3
ZNY10	0	2	0	2
ZNY42	-1	2	-1	2

ZOB29	0	2	1	2
ZSE48	0	1	0	1
ZTL15	1	2	1	2

An alternative approach for examining prediction accuracy as a function of prediction interval is to count the number of times the predicted number of aircraft were within a certain bound around the actual number of aircraft. Similarly, the number of times the predictions were above the bounds and below the bounds can be counted. These counts can then be used for computing the percentage of correct-prediction (within the bounds), under-prediction (below the bounds) and over-prediction (above the bounds). Figure 3 shows these results for Sector 93 (ZFW93) in the Fort Worth Center. Actual traffic-counts ± 2 aircraft were used as the bounds. The symbols on the graphs show the results at 15-minute, 30-minute, 45-minute, 60-minute, 90-minute and two-hour prediction intervals. The graphs show that the percentage of correct-prediction decreases with increasing prediction time intervals. The extent of under-prediction increases with increasing prediction time intervals. The degree of over-prediction remains small.

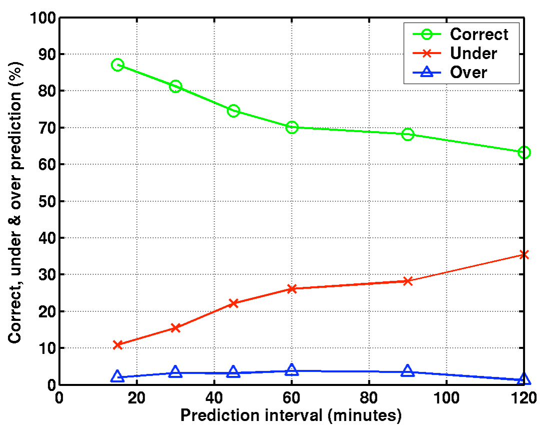

Figure 3: Percentage correct-prediction, under-prediction and over-prediction with respect to actual traffic-counts ± 2 aircraft for Sector 93 of the Fort Worth ARTCC.

These trends of decreasing correct-prediction and increasing under-prediction with increasing prediction time intervals were also seen for the other sectors in Table 1 and are summarized in Tables 4 and 5. The percentages (rounded off to integers) in columns two through seven are for 15-minute, 30-minute, 45-minute, 60-minute, 90-minute and two-hour predictions, respectively.

Table 4: Percentage correct-prediction within ± 2 aircraft.

Sector	15	30	45	60	90	120
ZAB70	89	87	84	79	78	76
ZBW10	81	76	75	73	70	62
ZDC12	74	73	73	71	71	70
ZDC72	67	65	55	60	61	62
ZDV18	78	74	70	68	63	61
ZFW48	68	71	70	71	69	56
ZFW93	87	81	75	70	68	63
ZHU26	91	89	86	86	82	82
ZID84	78	78	72	73	66	60
ZID98	59	60	58	59	58	64
ZKC84	74	70	64	62	62	61
ZKC98	82	78	71	71	67	69
ZLA30	69	69	71	67	65	61
ZLA35	72	61	62	61	58	53
ZMA59	69	69	67	65	63	63
ZNY10	54	61	59	63	67	69
ZNY42	70	74	73	71	69	60
ZOB29	75	72	69	64	66	66
ZSE48	96	95	94	93	92	90
ZTL15	70	70	65	65	67	66

Table 5: Percentage under-prediction.

Sector	15	30	45	60	90	120
ZAB70	6	10	11	18	20	21
ZBW10	6	7	13	16	19	32
ZDC12	3	6	11	10	13	20
ZDC72	5	8	7	12	16	23
ZDV18	19	23	25	25	31	34
ZFW48	8	19	19	22	25	43
ZFW93	11	16	22	26	28	35
ZHU26	5	7	9	9	15	15
ZID84	17	16	22	23	31	39
ZID98	2	5	9	10	12	13
ZKC84	17	20	22	24	25	34
ZKC98	5	7	9	10	15	20
ZLA30	9	29	26	31	33	38
ZLA35	26	38	36	38	41	46
ZMA59	31	31	33	35	37	37
ZNY10	2	7	7	9	10	23
ZNY42	7	15	19	23	26	38
ZOB29	11	13	16	15	20	25
ZSE48	4	5	6	7	8	9
ZTL15	5	8	12	12	15	22

This observation that the predicted traffic-counts are less than the actual traffic-counts for longer duration predictions, leads one to suspect that more aircraft are on the ground and flight plans are not available when predictions are made for longer time intervals. To ascertain the merits of this conjecture, percentages of airborne aircraft and those on the ground were computed as predictions were made during the chosen prediction time intervals. For example, consider the two-hour predictions shown at one-minute intervals in Figure 1. Numbers of aircraft that are airborne and that are scheduled to depart (currently on the ground) are counted in each two-hour prediction interval. Total numbers of airborne aircraft and those on the ground are obtained by adding all the two-hour prediction counts from the previous step. The percentages are then obtained by using these totals. The graphs in Figure 4 show the percentages of airborne and non-airborne aircraft that were predicted to be in Sector 93 of the Fort Worth Center as a function of 15-minute through two-hour prediction intervals. The two-hour data points were obtained as the prediction time history shown in Figure 1 was generated.

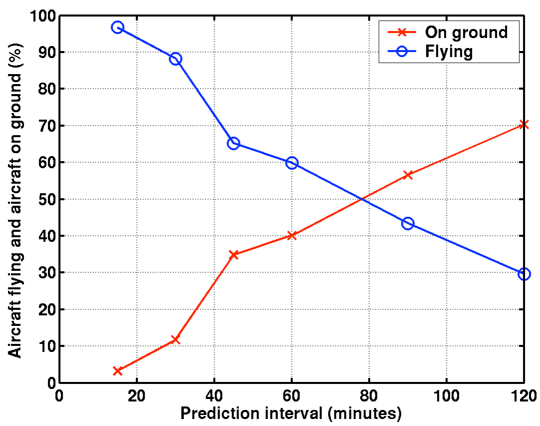

Figure 4: Percentages of airborne aircraft and aircraft on ground that are predicted to be in Sector 93 of the Fort Worth ARTCC as a function of prediction interval.

Observe from Figure 4 that the proportion of airborne aircraft to those on the ground decreases with increasing prediction time intervals. For example, 97% of the aircraft are airborne and only 3% are on the ground during a 15-minute prediction interval as compared to 30% are airborne and 70% are on the ground during a two-hour prediction interval. Since uncertainties are higher for aircraft on the ground and that there are more aircraft on the ground during longer prediction time intervals, the net result is that long-range forecast is not as accurate as short-range forecast.

The trends observed in Figure 4 for predictions of traffic-counts in Sector 93 of Fort Worth ARTCC were also seen for predictions of traffic-counts in all the other sectors listed in Table 1. These results are summarized in Table 6. The table shows the percentage of aircraft on the ground. To obtain the percentage of airborne aircraft, subtract the percentage of aircraft on the ground from 100. The values in the columns two though seven are for 15-minute through two-hour prediction intervals as indicated in the table header.

Table 6: Percentage of aircraft on the ground.

Sector	15	30	45	60	90	120
ZAB70	3	3	14	20	47	67
ZBW10	3	20	31	44	75	82
ZDC12	8	17	29	42	64	85
ZDC72	4	25	51	74	93	94
ZDV18	0	10	18	23	46	60
ZFW48	31	41	63	68	76	87
ZFW93	3	12	35	40	57	70
ZHU26	2	9	19	22	45	66
ZID84	11	43	56	65	86	94
ZID98	8	23	38	57	74	81
ZKC84	9	24	49	57	69	74
ZKC98	2	20	42	48	70	76
ZLA30	35	66	74	85	86	85
ZLA35	14	33	43	54	58	54
ZMA59	2	11	25	26	40	68
ZNY10	17	54	71	81	90	92
ZNY42	50	66	90	93	96	96
ZOB29	3	18	28	60	74	73
ZSE48	5	23	48	58	57	65
ZTL15	3	14	37	55	91	97

The next section describes a procedure for assigning probabilities to the traffic-count forecasts obtained using the deterministic procedure described in this section.

3. Probabilistic Prediction of Traffic-Counts

As results in the previous section show, there are differences in traffic-counts predicted by a deterministic algorithm compared to the actual traffic-counts. These errors were shown to increase with increasing prediction intervals. The conclusion that longer term prediction (beyond two hours when a large fraction of the aircraft in the prediction set are not airborne) is unreliable, causes difficulties from flow control perspective because flow control decisions are based on these predictions. The fact that the trajectory prediction process and the resulting traffic-counts will always be somewhat inaccurate due to modeling inaccuracies, departure-demand uncertainties, convective weather uncertainties, and lack of knowledge of flow restrictions, suggests that uncertainty bounds need to be provided around the predictions. A high probability of the predicted traffic-count exceeding the MAP value would then suggest that flow restrictions are immediately needed while a low probability would suggest that the decision to place restrictions could be postponed to a later time. Motivated by this objective, this section describes a method for computing the traffic-count probabilities.

Let, p_{ij} and p_{ej} be the probabilities of aircraft j entering a sector and leaving the same sector as a function of time. The probability p_j that the aircraft j is in the sector at any given time is obtained in terms of the entry and exit probabilities as:

$$p_j = p_{ij} - p_{ej} \qquad (1)$$

If n aircraft are predicted to be in the sector at some time, the probability that all these aircraft will be in the sector at the same time, P_n, is obtained by the product of the probabilities as:

$$P_n = \prod_{1 \le j \le n} p_j \qquad (2)$$

The probability that $n-1$ aircraft will be in the sector at the same time is given by:

$$P_{n-1} = \sum_{1 \le i \le n} q_i \prod_{\substack{1 \le j \le n \\ j \ne i}} p_j \qquad (3)$$

where,

$$q_i = 1 - p_i \qquad (4)$$

is the probability that aircraft i will not be in the sector. If three aircraft are predicted to be in the sector with probabilities p_1, p_2 and p_3, respectively, the probability of two aircraft being in the sector at the same time is obtained via Equation (3) as follows:

$$P_2 = q_1 p_2 p_3 + q_2 p_1 p_3 + q_3 p_1 p_2 \qquad (5)$$

Defining,

$$\alpha_j = \frac{q_j}{p_j} \qquad (6)$$

the expression in Equation (3) can be rewritten as:

$$P_{n-1} = P_n \sum_{1 \le j \le n} \alpha_j \qquad (7)$$

with P_n defined in Equation (2).

The probability that $n-2$ aircraft will be in the sector at the same time can also be computed, in a similar manner described earlier for the example in Equation (5), by considering all combinations of two aircraft not being in the sector at the same time as follows.

$$\begin{aligned} P_{n-2} = q_1 \sum_{2 \le i \le n} q_i \prod_{\substack{1 \le j \le n \\ j \ne 1 \\ j \ne i}} p_j + q_2 \sum_{3 \le i \le n} q_i \prod_{\substack{1 \le j \le n \\ j \ne 2 \\ j \ne i}} p_j \\ + \cdots + q_{n-1} q_n \prod_{\substack{1 \le j \le n \\ j \ne n-1 \\ j \ne n}} p_j \end{aligned} \qquad (8)$$

Using the definition in Equation (6), the expression for P_{n-2} can be rewritten as:

$$P_{n-2} = P_n \sum_{1 \le i \le n-1} \alpha_i \sum_{i < j \le n} \alpha_j \qquad (9)$$

A general expression for the probability of "m" aircraft being in the sector at the same time is obtained by construction as:

$$P_m = P_n \sum_{1 \le i_1 \le m+1} \alpha_{i_1} \sum_{i_1 < i_2 \le m+2} \alpha_{i_2} \cdots \sum_{i_{n-m-1} < i_{n-m} \le n} \alpha_{i_{n-m}} \qquad (10)$$

The probability that there will be m or more aircraft in the sector can now be obtained as:

$$R_m = \sum_{m \le i \le n} P_i \qquad (11)$$

Observe that $0 \le R_m \le 1$. The computed value of R_m can be compared with the predetermined Monitor Alert Parameter (MAP) to decide whether flow control initiatives are needed. For example, if the traffic-count (obtained using the deterministic forecasting procedure) is predicted to exceed the MAP value by one or two aircraft, flow restrictions are imposed following the currently used procedures. The decision would be quite different if the probability of exceeding the MAP value by two was less than 10%. The main benefit of the probabilistic approach is that it provides the decision maker with a metric of risk, which allows the decision maker to tradeoff the cost of allocating resources for highly unlikely events against the consequence of postponing the decision to a later time.

In addition to using the probabilities of individual aircraft for computing R_m via Equation (11), they can also be used for computing the average number of aircraft expected in the sector as a function of time. The probability, p_j, that aircraft j will be in a sector, implies that if a large number of trials (for example, L trials) were made, this aircraft would be in the sector $p_j L$ times. Although in reality, arrival of an aircraft into a sector may depend on the arrival of other aircraft due to traffic management initiatives such as metering or miles-in-trail, assuming that each aircraft arrives and departs independently of other aircraft results in:

$$N = L \sum_{1 \le j \le n} p_j \qquad (12)$$

aircraft in L trials. The average number of aircraft in a trail is then:

$$\overline{N} = \sum_{1 \le j \le n} p_j \qquad (13)$$

Thus, the average number of aircraft in a sector at a given time is obtained as the sum of the probabilities of individual aircraft.

The main results of the development presented in Equations (11) and (13) do not make any assumptions about how the probabilities of an aircraft arriving into a sector and departing from the sector, or being in the sector, are computed. For example, probabilities of entry into a sector and exit from the sector can be computed as a function of departure and arrival airports, route of flight, type of flight (long-haul or short-haul), distance from the sector, time of the day, weather conditions enroute and at the destination airport, and current or planned TFM initiatives. Such a model can be developed by analysis of historical air traffic and weather data. A much simpler model that only takes departure time uncertainty into account is described in the next section. This simpler model does illustrate the procedure for computing the probability of exceeding the sector monitor alert parameter value via Equation (11) when departure times of the aircraft are not known precisely.

4. Departure Time Uncertainty Modeling

To predict the time of arrival of an aircraft into a sector using a deterministic procedure, the time of departure $t_{d\,j}$ and the time of flight along the route of flight $t_{f\,j}$ are needed. The time of arrival at the sector is then obtained as:

$$t_{i\,j} = t_{d\,j} + t_{f\,j} \qquad (14)$$

The time of departure from the sector can be computed by adding the time it takes to fly through the sector. Thus,

$$t_{e\,j} = t_{i\,j} + T_f \qquad (15)$$

where, $t_{i\,j}$ is the sector entry time, T_f is the flight time though the sector and $t_{e\,j}$ is the sector exit time. Assuming that the flight time to the sector and also the time it takes to fly across the sector are modeled accurately, any uncertainty in the departure time directly translates into sector entry time and sector exit time uncertainties (see: Equations (14) and (15)).

The first source of departure time information for an aircraft is obtained from the airline schedule database in ETMS data. This information is updated when ETMS receives a flight plan message from the ARTCC Host computer. Between forty-five minutes to one and a half hours prior to departure, airline dispatchers file a flight plan with the air traffic control, which contains the proposed time of departure along with the aircraft type, cruise-speed, cruise-altitude and route information. Analysis of ETMS traffic data suggests that about 80% of the aircraft depart within +10 and –10 minutes of their proposed departure times [9]. About 92% of the aircraft depart within +20 and –20 minutes. Although the distribution of the departure delays fit Poisson distributions (a different one for departures from each airport) somewhat better than Gaussian distributions (see: Reference 13), Gaussian distributions have been used here for modeling departure time uncertainty because it is easy to obtain closed form expressions with them since Gaussian distributions are continuous.

As borne out by data analysis that most flights takeoff within a small interval (± 10 minutes) around the proposed time of departure, the mean of the departure distribution model can be set to the proposed time of departure. Adding the flight time to the mean of departure distribution model results in the mean of the sector entry distribution model as a consequence of Equation (14). In other words, the sector entry time distribution is the departure distribution shifted by the flight time needed for arriving at the sector. With the mean sector entry time t_μ given as:

$$t_\mu = t_d + t_f \qquad (17)$$

the probability density function for Gaussian (Normal) distribution model is:

$$f(t_i) = \frac{1}{\sqrt{2\Pi}\sigma} e^{-\frac{(t_i - t_\mu)^2}{2\sigma^2}} \qquad (18)$$

where, σ is the standard deviation of the departure delays (a characteristic of the airport from which this aircraft departed), and t_i is a sector entry time sample from the Gaussian distribution model. The probability of the aircraft having entered the sector at time t can be obtained by integrating Equation (18) from $-\infty$ to t as:

$$p_i = \int_{-\infty}^{t} f(t_i)\, dt_i = \frac{1}{2}\left[1 + erf\left(\frac{t - t_{\mu i}}{\sqrt{2}\sigma}\right)\right] \qquad (19)$$

The mean sector entry time $t_{\mu i}$ is given by Equation (17). Defining the mean exit time as:

$$t_{\mu e} = t_d + t_f + T_f \qquad (20)$$

the probability of departure from the sector can be computed by using the probability density function in Equation (18) as:

$$p_e = \int_{-\infty}^{t} f(t_e)\, dt_e = \frac{1}{2}\left[1 + erf\left(\frac{t - t_{\mu e}}{\sqrt{2}\sigma}\right)\right] \qquad (21)$$

The probability that the aircraft is in the sector at time t is obtained using Equations (1), (19) and (21) as follows.

$$p = \frac{1}{2}\left[erf\left(\frac{t - t_{\mu i}}{\sqrt{2}\sigma}\right) - erf\left(\frac{t - t_{\mu e}}{\sqrt{2}\sigma}\right)\right] \qquad (22)$$

Observe from Equation (22) that $p(-\infty) = 0$ and also $p(\infty) = 0$.

Once the probability of being in the sector is computed for every aircraft at time t, the probability that there will be more aircraft in the sector than that specified by the Monitor Alert Parameter can be determined using Equation (11) and the average number of aircraft (expected value) can be determined using Equation (13).

In order to use the probability computation procedure described in Equation (22), the standard deviation values of departure delay distributions are needed. Standard deviation values were computed for the top 40 airports in the United States by fitting a Gaussian model to the departure distribution data, following the procedure outlined in Reference 13. The standard deviation values in minutes are listed in Table 7. The average value of 16.5 minutes, which is the average of the standard deviation values of the 40 airports listed in Table 7, is used for departures from all other airports not listed in Table 7.

Table 7: Standard deviation of departure delay distributions for the top 40 U. S. airports.

Airport Code	City	σ_d
KATL	Atlanta	19.4
KBNA	Nashville	14.5
KBOS	Boston	16.5
KBWI	Baltimore	14.9
KCLE	Cleveland	18.5
KCLT	Charlotte	15.5
KCVG	Cincinnati	14.8
KDCA	D. C.	13.6
KDEN	Denver	18.5
KDFW	Fort Worth	21.5
KDTW	Detroit	18.4
KEWR	Newark	18.5
KFLL	Fort Lauderdale	15.6
KIAD	D. C.	18.5
KIAH	Houston	15.0
KIND	Indianapolis	19.2
KJFK	New York	19.6
KLAS	Las Vegas	14.5
KLAX	Los Angeles	12.2
KLGA	New York	22.3
KMCI	Kansas City	13.7
KMCO	Orlando	11.9
KMDW	Chicago	16.9
KMEM	Memphis	16.7
KMIA	Miami	15.2
KMSP	Minneapolis Saint Paul	16.8
KORD	Chicago	16.8
KPDX	Portland	15.0
KPHL	Philadelphia	21.5
KPHX	Phoenix	16.7
KPIT	Pittsburgh	16.6
KRDU	Raleigh Durham	15.6
KSAN	San Diego	12.6

KSEA	Seattle	17.1
KSFO	San Francisco	17.1
KSJC	San Jose	15.0
KSLC	Salt Lake City	15.7
KSTL	Saint Louis	15.8
KTEB	Teterboro	22.1
KTPA	Tampa	11.9

Starting with the proposed times of departures and the standard deviations of departure delay distributions for the various airports, the procedure for determination of probabilities of traffic-counts can be outlined via the following example in which traffic-count predictions are desired two hours in the future in Sector 93 of the Fort Worth ARTCC. Given the state information (position, velocity and route of flight) for aircraft that are currently airborne and intent information (proposed departure time, departure airport, type of aircraft, route of flight, cruise-speed and cruise-altitude) of the aircraft that are scheduled to depart in the next few hours, the deterministic forecasting procedure, described earlier in Section 2, is used to determine the estimated times of arrivals to Sector 93 and departures from Sector 93. The nominal entry and exit times computed using the deterministic procedure are then used as mean entry and exit times for the probability computation using Equation (22). Numerical values of the probabilities of the aircraft being in Sector 93 are then used in Equation (11) to determine the probabilities of "m" or more aircraft being in the sector at the same time. Plots of the computed probabilities are then used for guiding decision-making.

Figure 5 shows the probabilities of exceeding a certain number of aircraft in Sector 93 of Fort Worth ARTCC at particular instants of time. For example, the graph marked with the triangle symbol shows that the probability that there will be one or more aircraft in Sector 93 is 65%, two or more aircraft is 25%, three or more aircraft is 6% and four or more aircraft is 1%. Similarly, the graph corresponding to the probabilities at 17 hours and 30 minutes (noted as 17.5 in the legend) shows that the probability of exceeding four or more aircraft is 100%, five or more aircraft is 82%, six or more aircraft is 48%, seven or more aircraft in 20%, eight or more aircraft is 6% and nine or more aircraft is 1%. The envelope of the probability graphs, obtained using all time intervals, is marked with the circle symbol. This graph shows that at no time will Sector 93 have more than 14 aircraft. It also shows that at some instant of time there is a 40% probability that there will be ten or more aircraft in Sector 93. Both, the graphs at particular instants of time and the envelope, provide a measure of the likelihood that the sector will experience a certain traffic load. Thus, this technique provides a natural way for establishing confidence bounds. If the MAP value for Sector 93 were 7, the graph with the "x" symbol shows that there is only a 20% probability of seven or more aircraft, 6% of eight or more aircraft, and 1% of nine or more aircraft. Similarly, the envelope suggests that there is a 100% probability of seven or more aircraft, 91% of eight or more aircraft and 68% of nine or more aircraft being in the sector at some instant of time. The confidence bounds can be used by the decision-maker to commit the correct amount of resources balanced against the risk of exceeding the MAP value

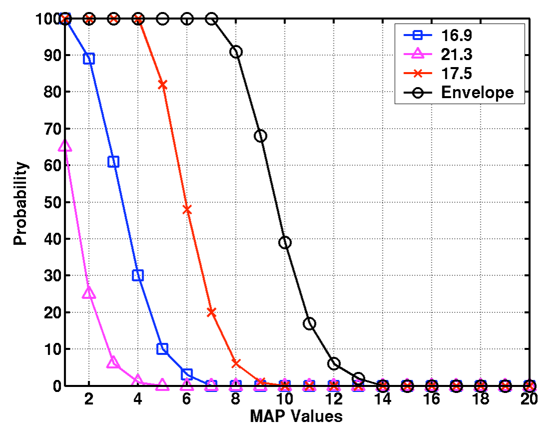

Figure 5: Probability of exceeding MAP values in Sector 93 of the Fort Worth Center at particular instants of time.

5. Conclusions

Due to the pivotal role of traffic-count forecasting for flow control decisions in air traffic management, this paper examined both deterministic and probabilistic techniques for predicting future traffic-counts in the sectors and confidence bounds on these predictions. The deterministic traffic-count procedure implemented in the Future ATM Concepts Evaluation Tool (FACET) was examined first. To compare the traffic-counts generated using the forecasting procedure with the actual traffic-counts in the sectors, a day's worth of air traffic data provided by the Enhanced Traffic Management System (ETMS) were recorded. Guided by past analysis of Enhanced Traffic Management System data quality, some aircraft were removed from these data during the preprocessing step for improving the data quality. Additional corrective steps were taken during runtime to compensate for data drops and reported altitude errors. The deterministic forecasting procedure used estimated groundspeed derived from ETMS data rather than model-based groundspeed for trajectory prediction, which is needed for computation of traffic-counts.

Algorithms in FACET were used for generating traffic-count predictions for twenty sectors in fourteen Air Route Traffic Control Centers in the continental United States at prediction intervals from 15 minutes through two hours. Statistics of the difference between the predicted and actual traffic-counts were presented for these twenty sectors. Accuracy of predictions was examined in terms of percentages of correct-prediction, under-prediction and over-prediction as a function of prediction intervals. These results were found to be correlated to the decreasing percentages of airborne aircraft as a function of increasing prediction time intervals.

Building on the deterministic traffic-count forecasting procedure described in the first part of the paper, the method for probabilistic prediction of traffic-counts is examined in the second part of the paper. The general expression for computing the probability of having a certain number of aircraft or more in the sector at any given time was derived. Gaussian model of departure uncertainty was assumed for deriving expressions for computing the probability that an aircraft will be in a sector. A numerical example was presented to illustrate the method for computing the probabilities of exceeding pre-established traffic-count thresholds and using them for decision-making.

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